

# ACTIVE VIBRATION ISOLATION IN A “SMART SPRING” MOUNT<sup>1</sup> USING A REPETITIVE CONTROL APPROACH

S. Daley, J. Hätönen & D. H. Owens

*Department of Automatic Control & Systems Engineering, University of Sheffield, Sheffield, S1 3JD, UK.*

**Abstract:** In a variety of different engineering systems there is a requirement to isolate sensitive equipment from foundation vibration or alternatively, isolate the foundation from machinery vibration. Passive solutions to this problem provide some isolation but performance is significantly degraded in the presence of structural compliance. A recently proposed hybrid active/passive solution known as the “Smart Spring” mounting system specifically addresses this problem of compliance. In earlier work on this system the required local controller was based on LQG design on the assumption that the vibration sources are random. The work reported here investigates the application of a repetitive control approach to deal with periodic vibration sources. The industrial potential of the approach has been shown using an experimental facility where isolation results in the region of 50dB have been achieved. *Copyright © 2005 IFAC*

**Keywords:** Active control, Marine Systems, Repetitive Control, Adaptive Algorithms

## 1. INTRODUCTION

Vibration that is transmitted to the foundation from machinery or to sensitive equipment from seismic events can be problematic in a large variety of applications. Traditionally, the problem of vibration propagation is tackled by isolating the source by mounting the machinery or sensitive equipment on a set of resilient elastomeric mounts (Crede, 1951). These passive mounts provide some degree of vibration isolation, however, performance markedly deteriorates in the presence of structural compliance. Moreover, passive design for low frequency is difficult and often represents a compromise between isolation performance and supported machinery alignment.

As a result of this limitation in passive systems design, there has been considerable research activity devoted to developing active and semi-active schemes for vibration isolation over the last three decades (see for example Fuller et al, 1996). Example applications include isolation in aircraft (Swanson and Miller, 1993), automobiles (McDonald et al, 1991), wafer production (Anderson

& Houghton, 2001) and Naval systems (Winberg, et al, 2000). In the marine environment, vibration that propagates from propulsion and auxiliary machinery can cause significant problems associated with passenger and crew comfort and also through the generation of acoustic noise from the hull. Such acoustic noise creates a severe detection hazard in Naval vessels and is also problematic for civil vessels such as those used by fisheries research organisations.

As a result of the restricted performance of passive systems and the increasingly demanding vibration requirements, alternative designs based on active control technology continue to be sought. In the UK during the 1990's a programme led by GEC-Marconi (now BAE Systems) and funded by the US Office of Naval Research led to the development of a fully active solution (Johnson & Swinbanks, 1996, Darbyshire & Kerry, 1997, Daley, 1998). This was based upon electromagnetic levitation of the machinery raft, however, the original concept required large numbers of actuators and therefore represented an expensive solution.

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<sup>1</sup> “Smart Spring” is a trademark of BAE Systems



Figure 1: Prototype Actuator for a “Smart Spring” Mounting System

More recently, BAE Systems have been developing an active isolation technology known as the “Smart Spring” mounting system<sup>2</sup> (Daley et al, 2004, Daley and Johnson, 2004). This is a hybrid active/passive solution (figure 1) that is a more practical and lower cost development of the fully active technology. In the work reported previously, the local mount controller was based upon a modal LQG solution that had a feedforward and high gain feedback component. In the current paper, an alternative approach based upon a recently proposed repetitive control algorithm is presented. Moreover, a more realistic design that incorporates a standard passive mount and enables 6 degree of freedom control is utilised in the experimental programme.

## 2. REPETITIVE CONTROL

Repetitive Control (RC) has recently emerged as an important design approach for dealing with systems that are characterised by periodic behaviour. The basic and defining philosophy is to utilise information obtained during previous periods to improve current performance through a process of learning. Since first proposed in 1981 (Inouye et al, 1981), an increasing number of diverse applications of RC have been reported in the literature during recent years. Examples include, robotics (Kaneko and Horowitz, 1997), motor control (Kobayashi et al, 1999), rolling mills (Garimella and Srinivasan, 1996) and vibration control (Hillerström, 1996).

The RC algorithm utilised here (from Hätönen et al 2004) is derived from the assumption that the plant can be represented by the discrete time model

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k) \\ y(k) &= Cx(k) \end{aligned} \quad (1)$$

where  $\Phi, \Gamma, C$  and  $D$  are matrices of appropriate dimensions,  $k \in [0, 1, 2, \dots, \infty)$  and for ease of exposition a plant relative degree of 1 has been

<sup>2</sup> This is the subject of several Patents.

assumed. It is further assumed that the system is stable, controllable and observable.

The design aim generally considered in RC is to make system (1) track a reference signal,  $r(k)$ , as closely as possible (i.e.  $\lim_{k \rightarrow \infty} e(k) = 0$ ;  $e(k) \triangleq r(k) - y(k)$ ). The distinction from conventional control design is that the reference is also known to be  $N$ -periodic such that  $r(k) = r(k+N)$ <sup>3</sup>.

If the plant model is alternatively expressed as a function of the backward shift operator,  $q^{-1}$ , as follows

$$y(k) = G(q)u(k) = C(qI - \Phi)^{-1}\Gamma u(k) \quad (2)$$

and this has a finite impulse response (FIR) that converges to zero in less than  $N$  steps, then an algorithm<sup>4</sup> that satisfies the design goal is

$$u(k) = q^{-N}u(k) + \beta G(q^{-1})q^{-N}e(k) \quad (3)$$

The FIR assumption ensures both the causality of the algorithm (i.e.  $u(k)$  is only then dependent on *previous* values of the error up to order  $k-N$ ) and convergence provided that

$$\sup_{\omega \in [0, 2\pi]} \left| 1 - \beta \left| G(e^{j\omega}) \right|^2 \right| < 1 \quad (4)$$

However, as noted in Hätönen *et al* (2004), it is rare for a system to have a FIR and so it is intuitive to approximate the infinite impulse response (IIR) system via an appropriate truncation. By rewriting the plant model as

$$y(k) = G(q)u(k) = G_o(q)U(q)u(k) \quad (5)$$

where  $G_o(q)$  is used to approximate the true plant but satisfies the FIR assumption, and  $U(q)$  is a multiplicative uncertainty term used to represent the error due to the truncation (or any other modelling error), Hätönen *et al* (2004) were able to establish the practically important result that algorithm (3) still converges provided that the condition

<sup>3</sup> Note that in this paper the problem is one of output disturbance rejection and the reference is zero. However, on the assumption that  $d(k) = d(k+N)$  then the error (or in this case, output) evolution equation is identical and so the algorithm convergence properties remain unchanged.

<sup>4</sup> The algorithm uses the notation, that if

$$f(q) = f_0 + f_1q^{-1} + f_2q^{-2} + \dots$$

then

$$f(q^{-1}) \triangleq f_0 + f_1q + f_2q^2 + \dots$$

$$\sup_{\omega \in [0, 2\pi]} \left| 1 - \beta U(e^{j\omega}) \left| G(e^{j\omega}) \right|^2 \right| < 1 \quad (6)$$

is satisfied. Further, the authors also showed that condition (6) could always be met with a suitably small  $\beta$  if the phase of the nominal model,  $G_o(q)$ , lies within a  $\pm 90$  degree boundary around the phase of the true plant, . It is this approach that is adopted in the vibration isolation design study presented in the following sections.

### 3. EXPERIMENTAL FACILITY

The facility used for the study (figure 2) was originally developed in association with BAE Systems Marine during the late 1980's. The main purpose of this mount is for testing active isolation schemes for large marine machinery rafts. The active mount consists of a central standard passive elastomeric Naval mount around which are located 6 Ling 30N electro-dynamic shakers. These apply forces in parallel to the passive mount and the "stinger" attachments are arranged in a hexapod or Stewart platform style such that control can be applied to all six degrees of freedom (three orthogonal translational forces and three orthogonal torques).

In a practical installation such mounts would be distributed under the machinery or machinery raft. However, since a single mount is statically stable, it can be used in isolation to test local control strategies. A Gearing and Watson 170N inertial shaker is located on top of the mount to replicate the disturbance forces that would be encountered in practice. In earlier work on the "Smart Spring" mounting system (Daley & Johnson, 2004) a load cell was integrated into the mount in order to measure transmitted force. Here, the vibration transmission is monitored using six below mount accelerometers, located at the base of each control shaker.



Fig. 2: Experimental active mount

At its most complex, the control system has six inputs (control shaker drive voltage) and six outputs (accelerometer signals). The magnitude of a typical frequency response function for the control channels is shown in Figure 3 (the third accelerometer output to the third control shaker input). The fundamental (axial) mount resonance at 60Hz is clearly displayed.

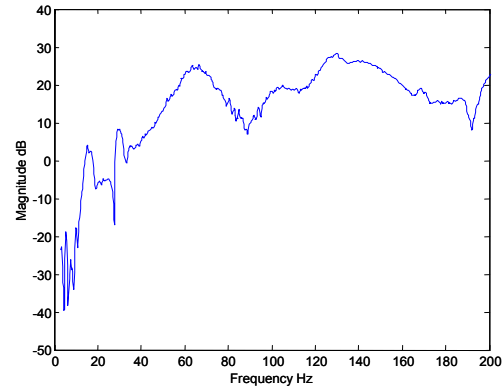


Fig. 3: Magnitude response of the third accelerometer to the third control shaker

The magnitude of the frequency response function for the same accelerometer and the disturbance shaker drive signal is shown in Figure 4. Again the 60Hz mount resonance is evident. in a full installation this would occur at a lower frequency due to the additional mass loading. A clear resonance at 20Hz is also shown, which is due to the internal suspension of the shaker.

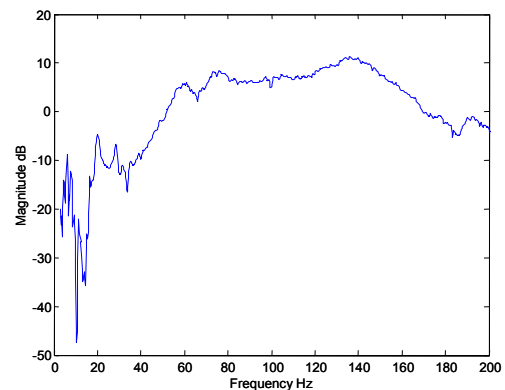


Fig. 4: Magnitude response of the third accelerometer to the disturbance shaker

For an active mount, the fundamental control aim is to minimise the base acceleration for a particular above mount vibration force. As the focus of the study reported here was for isolation in marine systems, the requirement is to tackle the vibration induced by main propulsion machinery and power units. For such systems, the predominant prime mover is a turbine or diesel where the vibration is characterised by discrete frequencies at multiples of the rotational speed. As a result, the work to date has concentrated solely on discrete frequency control. Some preliminary results from the application of the repetitive control approach outlined in section 2 are presented in the following section.

#### 4. EXPERIMENTAL RESULTS

To reduce the complexity of the initial work, control was only considered in the axial direction of the mount. In practice, in marine environments, this is the most problematic path due to coupling with hull natural modes. Fortunately, due to the hexapod arrangement, the control problem can be readily reduced to an equivalent SISO system in the axial direction by summing the acceleration signals and providing a common drive to all control shakers. The resulting control and disturbance path transfer functions are shown in figures 5 and 6 respectively.

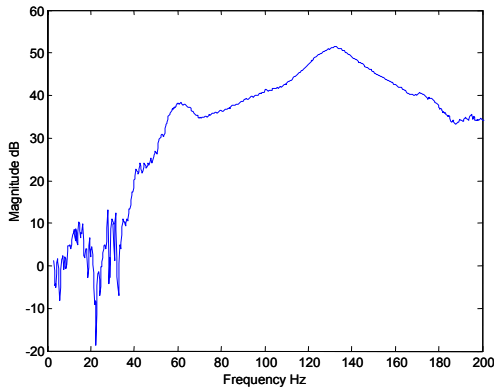


Fig. 5: Frequency response of the axial path control channel

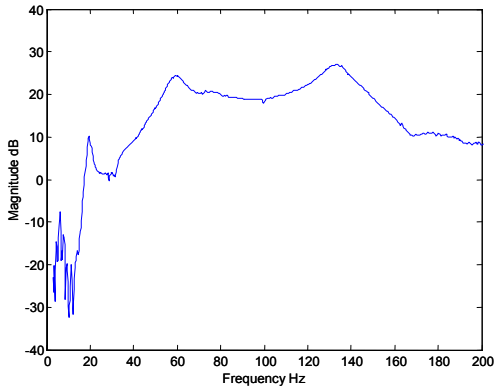


Fig. 6: Frequency response of the axial path disturbance channel

Implementation of algorithm (3) requires the impulse response of the plant (figure 7). Since this is difficult to obtain practically due to the characteristics of the control shakers, it was derived using the inverse Fourier transform of the frequency response function (figure 5).

The maximum force transmission occurs at the mount resonance and so the experiment was undertaken using a discrete excitation signal at 60Hz. For the chosen sampling interval of 0.0025 seconds, the minimum value for  $N$  (see section 2) is 20, representing 3 cycles at 60Hz. However,  $N$  was selected as 40 (i.e. 6 cycles) to reduce the magnitude of the modelling error caused by truncation. was

therefore chosen as the first 40 points in the impulse response of figure 7.

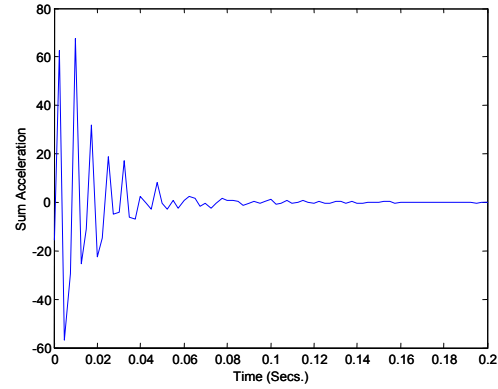


Fig. 7: Control channel impulse response

The phase for this nominal plant model together with the true plant are shown in figure 8., The only significant errors occur at low frequency, however, since the nominal phase remains within 90 degrees of the true plant, condition (6) can be satisfied.

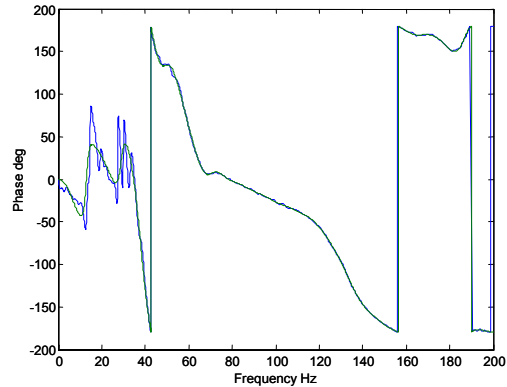


Fig. 8: True plant and truncated model phase characteristics

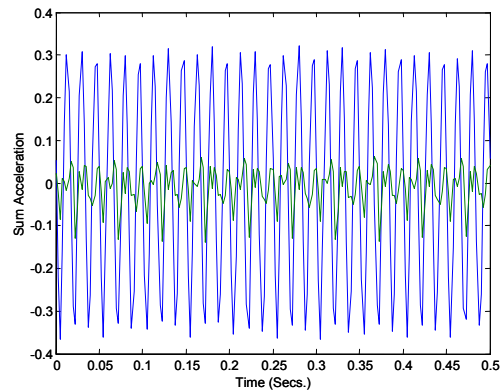


Fig. 9: Time response of the sum acceleration: with and without control

The result of applying algorithm (3) (following convergence) is shown in figures 9 (time series) and 10 (power spectrum). Although this represents a good performance it can be seen from figure 10 that the degree of isolation is limited to 20dB at 60Hz. Moreover the control action leads to significant components at a fundamental frequency of 20Hz (disturbance shaker resonance) and its harmonics.

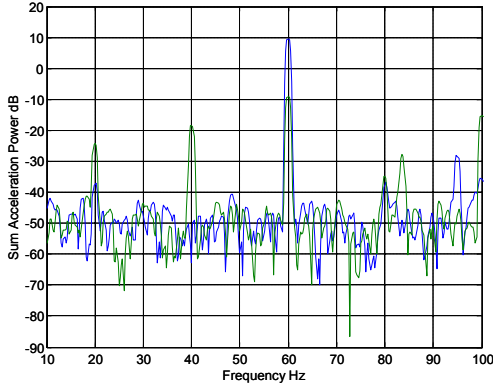


Fig. 10: Power spectral density of the sum acceleration: with and without control

The limit in performance is caused mainly by saturation in the control shakers. In order to more efficiently target the 60Hz disturbance frequency, algorithm (3) was modified to incorporate a band-pass filter  $G_f(q)$  as follows

$$u(k) = G_f(q)G_f(q^{-1})q^{-N}u(k) + \beta G(q^{-1})q^{-N}e(k) \quad (7)$$

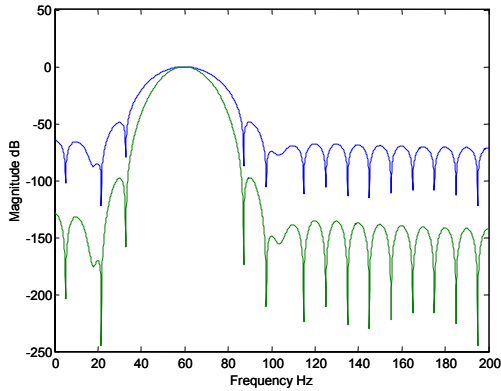


Fig. 11: Magnitude characteristics of  $G_f(q)$  and  $G_f(q)G_f(q^{-1})$

An FIR filter with corner frequencies of 50 and 70 Hz is used as shown in figure 12. The filter is selected as 40th in order to ensure the causality of the algorithm. Also shown in the figure are the magnitude characteristics of  $G_f(q)G_f(q^{-1})$ . This clearly demonstrates that implementation in this way has the advantage of sharpening the filtering action. It will also be noted that this combined filter has zero phase shift at all frequencies, a characteristic that is only possible by exploiting the periodic nature of the system.

The results of applying algorithm (7) to the active mount are shown in figures 12 and 13. It can be seen that the addition of the filter has resulted in a reduction of below mount acceleration of the order of 50dB. This represents an extremely good isolation performance. It will also be noted that although the fundamental disturbance shaker resonance at 20Hz is

still present, the first harmonic at 40Hz has been reduced.

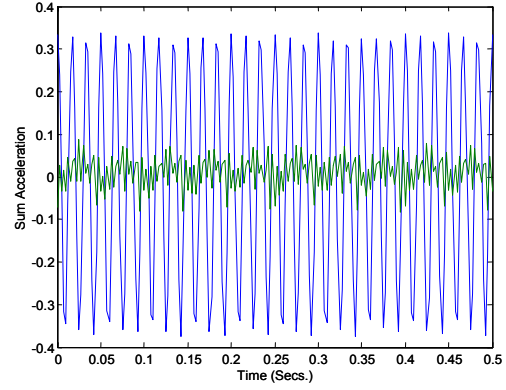


Fig. 12: Time response of the sum acceleration: with and without control

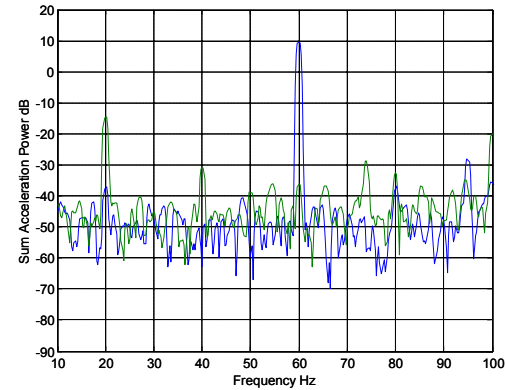


Fig. 13: Power spectral density of the sum acceleration: with and without control

It is thought that the 20Hz component could be explained either by an error in the original frequency response measurement (due to the resolution) or by uncontrolled excitation of the disturbance shaker. Development of an approach to target this additional component is the subject of ongoing research as is the comparison of the RC results with more established disturbance decoupling design methods.

## 5. CONCLUSIONS

The work presented here has demonstrated the industrial potential of a recently developed repetitive control algorithm. The basic algorithm together with a filtered variant has been applied to control in an active vibration isolation mount. The modified algorithm, that incorporates a zero phase shift filtering action that exploits the periodic nature of the problem, achieved isolation of 50dB at the targeted frequency. This performance was achieved using an efficient and practical design procedure in which all the necessary parameters were derived from frequency response measurements.

## 6. ACKNOWLEDGMENTS

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